①

姜胜英

Topological phases in tensor network

≡

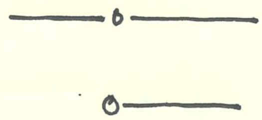
≡ degeneracy : finite

- tensor and its representation
- 1D MPS & topological phases:
- fermionic tensor
- 1D FSPT — Majorana chain; $\mathbb{Z} - \mathbb{Z}_8$
- 2D PEPS & topological phases:

Graphics

$\hat{O}$: matrix

$|\Psi\rangle$:

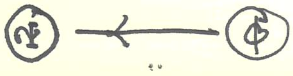


$\otimes$:

tensor network

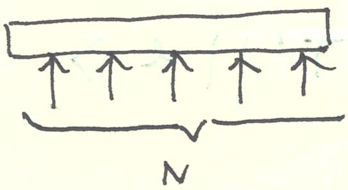
②

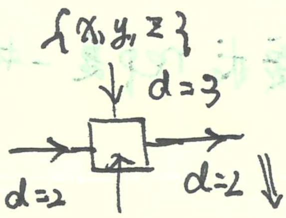
$|\Psi\rangle \triangleq$  ket $\langle\Psi| =$ 

$\langle\Phi|\Psi\rangle =$ 

$\hat{O} = O_{ij}|i\rangle\langle j|$ $\hat{O}|\Psi\rangle =$ 

- spin-1/2 chain

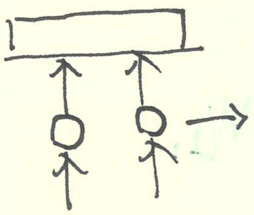
$|\Psi\rangle =$ 



$\{\sigma_x, \sigma_y, \sigma_z\}$

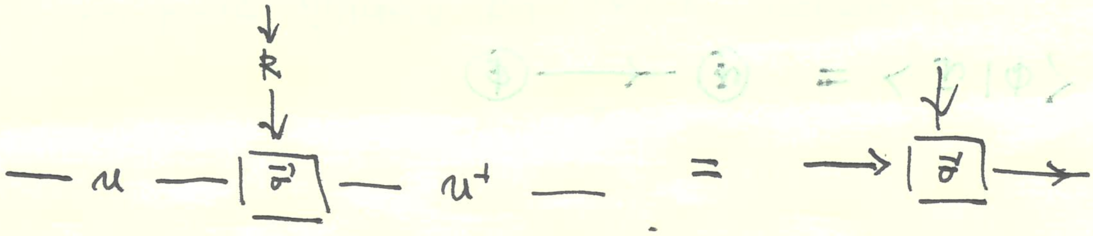
$\Downarrow$

$(|\uparrow\rangle, |\downarrow\rangle)$

-  $\rightarrow e^{\frac{i}{2}\vec{\sigma}\cdot\vec{n}}$

$H = -J\sum Z_i Z_{i+1} - h \sum X_i$

③

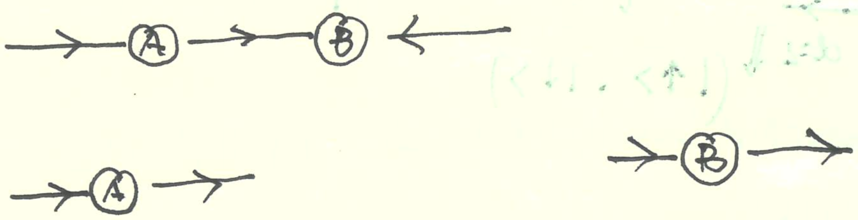


spin 1 : $SO(3)$

$$\langle l, m \,|\, T^{1}_{\alpha} \,|\, l, n \rangle = C^{l,m}_{l,n;1,\alpha} \langle l \,\|\, T^{1} \,\|\, l \rangle$$

contraction

缩并要求 rep 是一样的



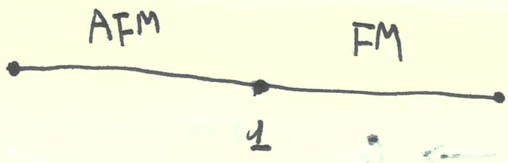
$$W_g \rightarrow A \rightarrow W_{2g} - W_2 - B \rightarrow W_{g_1}$$

T_{uv} $\qquad$ T_{vp}

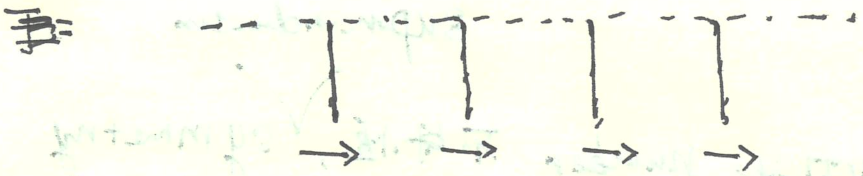
$\Rightarrow T_{uv} T_{vp}$

④

$$H = -J\sum_j Z_j Z_{j+1} - h\sum X_i$$



$J=0$: $|\rightarrow \; \rightarrow, \rightarrow, \rightarrow \rangle$



Z_2: symmetry: $g = \prod_j X_j$ $\quad [g, H] = 0$

$J \to \infty$: $|\uparrow\uparrow\uparrow \cdots \uparrow\rangle$ $\quad |\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$

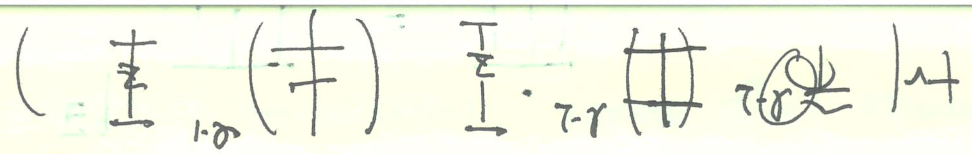
even: $\frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)$ ①

odd: $\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$ ②

在 Finite system 下: ① 和 ② 不是 strictly degenerate

the gap has: $\Delta = e^{-\xi/L}$

"ODLO"



$= \langle \sigma^z \sigma^z \rangle$

⑨

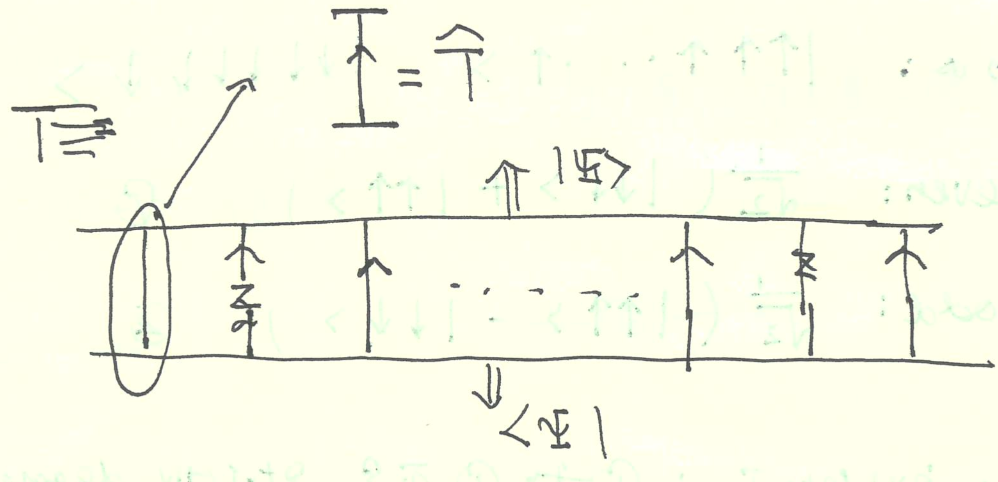
⑤

"ODLO" measure long range correlation of order parameter.

$$\lim_{|i-j|\to\infty} \langle Z_i Z_j \rangle = \begin{cases} \to 0 \\ \to C \neq 0 \end{cases}$$

superconductum

U(1) system particle number 不守恒, symmetry broken的 problem. (这个我还是不明白)

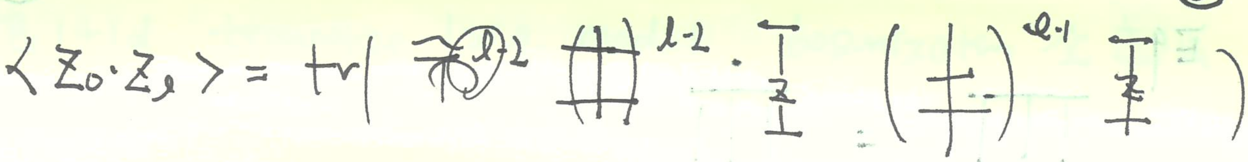


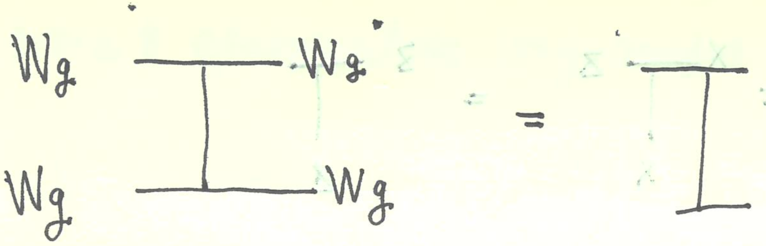
$$\boxed{T W_g = - W_g T}$$

Decomposition: $\hat{T} = \lambda_1 |r_1\rangle\langle r_1| + \cdots$

"QTO" measure long range correlation of order

5





• 为什么可以写出 MPS wavefunction

TN — sym

tensor Eq

• $Z_2 \otimes Z_2$ SPT ZXZ model

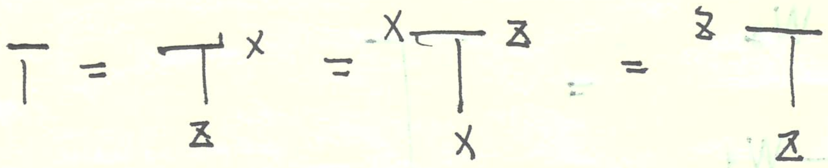
$Z_2 \otimes Z_2$

$$H = -\sum_{j} Z_{j-1} X_j Z_{j+1} \qquad \prod X_{2j} \quad \prod X_{2j-1}$$

$$\Rightarrow [\, Z_{j-1} X_j Z_{j+1},\ Z_{j'-1} X_{j'} Z_{j'+1} \,] = 0$$

$Z_{j-1} X_j Z_{j+1}$

Eq



- Zhong Long

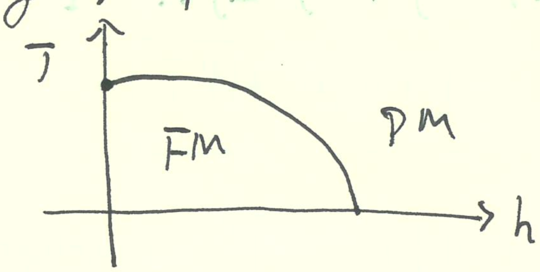
Ising models: Quantum / classical

transfer matrix

RG / idea

Fixed points: scaling behavior

- ~~One dimensional Ising model exact solution~~
- topological phases in Ising model



④ 63

⑧

把1+1d transverse Ising model bosonization会怎么样?

为什么是classical的Ising model?

因为每一个configuration $\{S_i\}$都是H的本征态

$$H = -\sum_l \sigma_l^z \sigma_{l+1}^z \cdot J - h\sum_l \sigma_l^x$$

· solution:

$$Z = \sum_{\{S_i\}} e^{K S_l \cdot S_{l+1}} = \sum_{\{S_i\}} T_{S_l \cdot S_{l+1}} \cdot T_{S_{l+1} \cdots}$$

$$\cdots T_{S_l \cdot S_1} = tr(T^l)$$

Wher $T = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix} = e^K + e^{-K}\sigma_x$

$$e^K \pm e^{-K} = \begin{cases} 2\cosh K \\ 2\sinh K \end{cases}$$

$$\Rightarrow (2\cosh K)^l + (2\sinh K)^l = Z$$

⑨

$$F = -k_B T \log 2 \cdot N - k_B T \log\left(\cosh^N K + \sinh^N\right)$$

correlation function:

$$\langle S_i \cdot S_j \rangle = \frac{\sum_{\{S_i\}} S_i \cdot S_j \, e^{K S_l \cdot S_{l+1}}}{\sum_{\{S_i\}} e^{K S_l \cdot S_{l+1}}}$$

$$= S_0 \cdot T \cdot T \, S_{i-j} \, T^{l-i-j}$$

$$S_0 T = S_0(\sigma + \sigma_x \alpha) = S_0(\sigma + \sigma_x \cdot \sigma)$$

$$= \mathrm{tr}\left(T^{l-(i-j)} \, \bar{T}^{i-j}\right)$$

$$= \mathrm{tr}\left(T^{l-2(i-j)} \times (e^{2K} - e^{-2K})\right)$$

$$\simeq (2\sinh 2K)^{i-j} \, (\cosh 2K)^{l-i-j}$$

$$\Rightarrow \langle S_i S_j \rangle = (\tanh 2K)^{i-j} = e^{-|i-j|/\xi}$$

$$\Rightarrow 1/\log(\tanh 2K) \Rightarrow$$

- 都是一个 finite correlation 的体系

  当 K→0, correlation length 会发散的

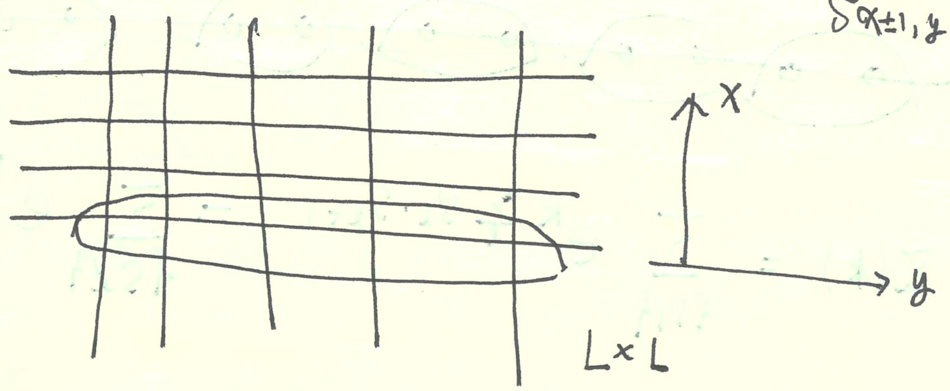
- Exact solution of two-dimension Ising model:

$$\Rightarrow \sum_{y} S_{x\pm1,y\pm1} S_{x,y}$$

$$Z = \sum_{\{S_{x,y}=\pm1\}} e^{K\sum_{x,y}\left(S_{x\pm1,y}\cdot S_{x,y} + S_{x,y\pm1} S_{x,y}\right)}$$

$$\sum_{x} \cancel{S_{x\pm1,y\pm1}}$$

$S_{x\pm1,y}$



$$\cancel{= \sum \left(T_{S_1,S_2,\cdots\cdots S_{yL}}\right)} \; ($$

$$= \sum \prod_{y} T^{S_1^y \cdots\cdots S_1^y}_{S_1^{y\pm1} \cdots\cdots S_1^{y\pm1}} \quad \prod_{i=1}^{\infty} T \quad S_N^{y} : S_N^{y-1}$$

$$T^{S_1^y}_{S_1^y} = \cosh K + \sin K \sigma_x$$

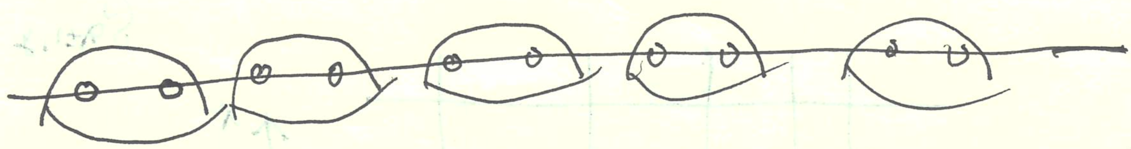
$$T = (\cosh K + \sin K \sigma_x) \otimes (\cosh K + \sin K \sigma_x) \cdots\cdots$$

$\Rightarrow \cancel{\;\;} = = (\mathrm{ch}\,k)^2 \cdot 2 \cdot \dfrac{e^{k\cdot S_3 \cdot S_4}}{\mathrm{ch}\,k}$

• term: $\sum_x S_{x,y\pm1}\cdot S_{x,y}$

$$T' = \bigotimes_x \left(\cosh k + \sinh k\, \sigma_x\right)_x \, e^{\frac{1}{2}k\,\sigma^3_x \cdot \sigma^3_{x+1}}$$

• Duality transformation



$$Z(k) = \sum_{\{S_l\}} e^{k\sum_l S_l\cdot S_{l+1}} = \sum_{\{S_l'\}} e^{k'\sum_l S_l'\cdot S_{l+1}'}$$

$$k \to \frac{2\pi}{L}$$

$$\sum_{S_3=\pm1} e^{kS_2\cdot S_3}\cdot e^{k\cdot S_3\cdot S_4} = (\cosh k)^2 \left(1 + \tanh k\cdot S_2\cdot S_3\right)\left(1+\tanh k\cdot S_3\cdot S_4\right)$$

$$= 2\cosh^2 k\left(1+\tanh^2 k\cdot S_2\cdot S_4\right)$$

$$\Rightarrow \cancel{chK} = \quad = (chK)^2 \cdot 2 \cdot \frac{e^{K' \cdot S_2 \cdot S_4}}{chK}$$

$$Z[K] \overset{block}{=\!=\!=} Z[K'] \left(\frac{2chK^2}{chK'}\right)^{L^2}$$

⇓ 每做一次出现一次

$$(thK' = th^2 K)$$

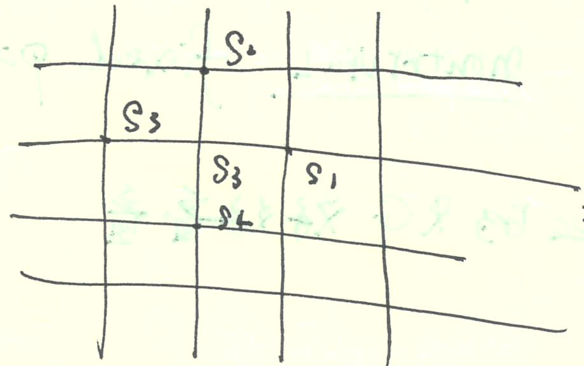
Fix point: 在RG变换下不变的

0:(~~Fixed~~ infinite)　　1 (zero temperature)

在长波极限下都等价于无穷高温

$$\xi(x) = \xi(x^2) \cdot 2$$

$$\Rightarrow \xi(x) = \frac{c}{\operatorname{Log} x} \quad (\text{一致})$$



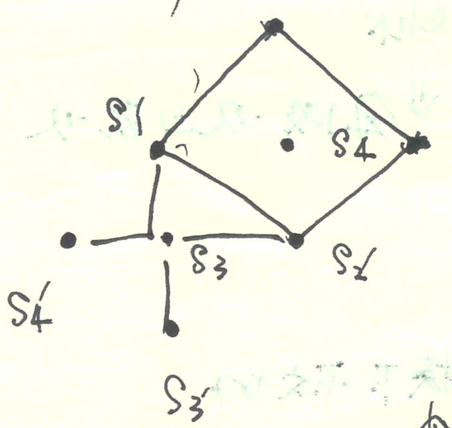
$$\sum_{S_3 = \pm 1} e^{K S_1 S_3} \ldots\ldots$$

$$= 2 (chK)^4 (1 + S_3' S_1 + S_3 S_2' + S_1 \cdot S_2 + S_1 \cdot S_2 + S_2 \cdot S_4) + S_1 S_2 \cdot S_3' \cdot S_4) \tanh K)$$

keep NN interaction

$$\simeq \quad 2(\mathrm{ch}K)^4\Big(1+(\mathrm{ch}K)^2(S_1'\cdot S_2'+S_1'S_4'+S_3'\cdot S_4'$$

$$+S_1'\cdot S_3')\Big)$$

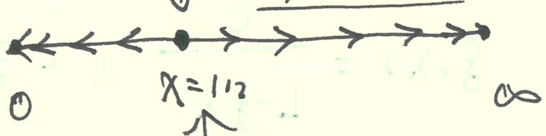


$$\cong 2(\mathrm{ch}K)^4\Big(1+\overset{\mathrm{ch}K'}{\overset{\Uparrow}{\boxed{2\mathrm{ch}^2K}}}(S_1'\cdot S_2' \dots)\Big)$$

$$\cong Z[K']\cdot\frac{(2\mathrm{ch}^4K)^{N/2}}{(\mathrm{ch}K')^{N/2}}$$

RG的 unstable 的 fixed point 区分了两个 phase.

$$x'=2x^2$$



nontrivial fixed point

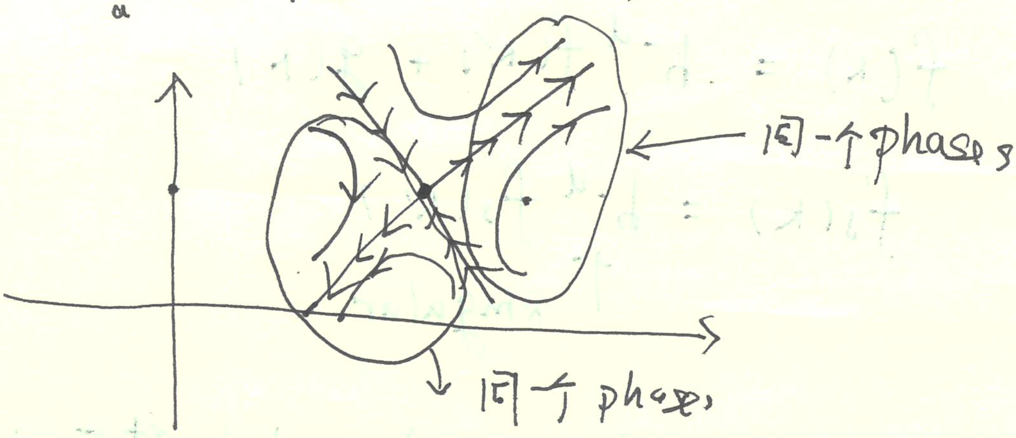
回去把 momentum space 的 RG 好好看看

• 在高维的cases下

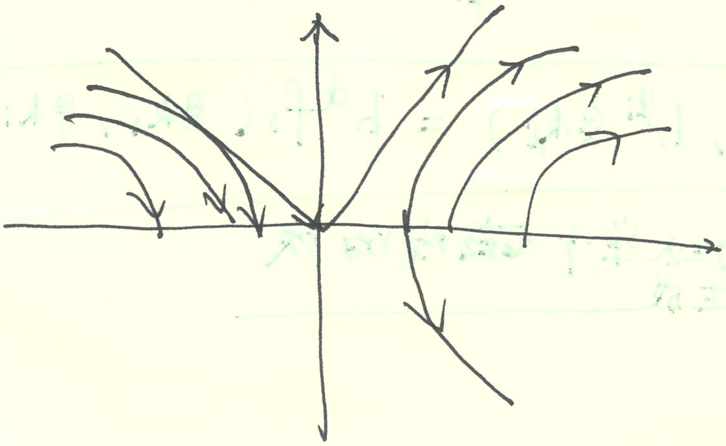
在Fixed point附近做perturbation expansion

$$(\delta k')_a = \sum_b T_{ab} (\delta k)_b$$

$$\sum_a T_{ab} \phi_a^{(i)} = \lambda^{(i)} \phi_a^{(i)}$$



perturbation expansion看RG fixed point上的 RG flow.



irrelevent / relevent / marginal

#

$$Z(k) = Z(k') \times \text{analytical}$$

$$e^{-L^d f(k)} = e^{-(L/b)^d f(k')} e^{-L^d g(k)}$$

$$\Rightarrow \quad f(k) = b^{-d} f(k') + g(k)$$

$$\Rightarrow \quad f_s(k) = b^{-d} f_s(k')$$

↑ singular

$\delta k_i' = \lambda_i \delta k_i$ $\quad \lambda_1 > 1, \ \lambda_i < 1$ (对应二级相变)

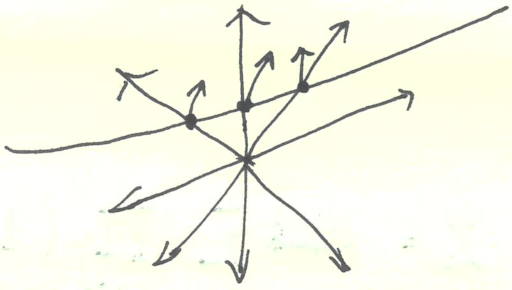
$\lambda_i = b^{y_i}$ $\quad y_1 > 0, \ y_i < 0$

$$\boxed{f_s(b^{y_1}\delta k_1, b^{y_i}\delta k_i) = b^d f_s(\delta k_1, \delta k_i)}$$

为什么加入单个磁场的项
生成

$\{$ h ← 通常是relevent的

irrelevant / relevant / marginal



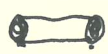
· Duality of 1D Ising model

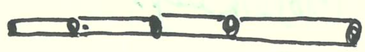
$$Z = \sum_{\{S_\ell\}} (\mathrm{ch}k)^{\ell} \left(1 + S_\ell \cdot S_{\ell+1} \tanh k \right) \quad ①$$

$$\sum_{S_\ell = \pm 1} \left(1 + S_{\ell-1} \cdot S_\ell \tanh k\right)\left(1 + S_\ell \cdot S_{\ell+1} \tanh k\right)$$

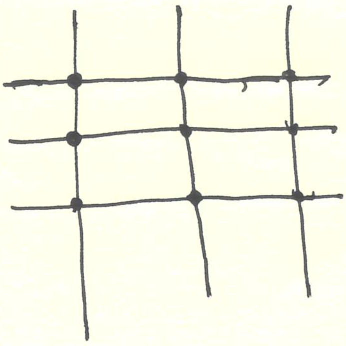
$$= 1 + S_{\ell-1} \cdot S_{\ell+1} \tanh^2 k$$

1 : 

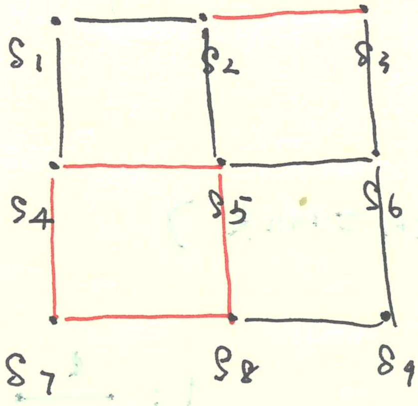
$S_\ell \cdot S_{\ell+1}$ 

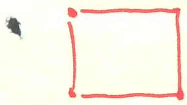
$$① = (\mathrm{ch}k)^2 \Big($$  $$+$$  $$\Big)$$

Generalized into 2D Ising model:



$$Z = \cancel{\mathrm{ch}\,k} \sum_{\{S_i\}} \prod_{\langle ij \rangle} (1 + S_i \cdot S_j \tanh k) \cdot \cosh k$$





$$(S_4 \cdot S_5 \cdot S_5 \cdot S_8 \cdot S_8 \cdot S_7 \cdot S_7 \cdot S_4)\,(\tanh k)^4$$

$$Z = (\mathrm{ch}\,k)^{2L^2} \sum_{\text{closed loops}} (\tanh k)^{\text{Length of loops}}$$

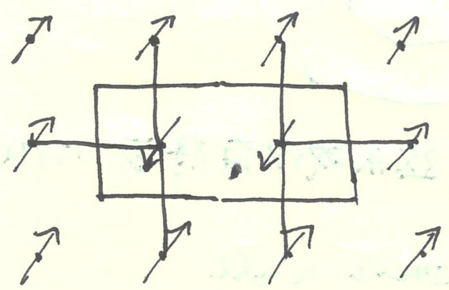
$\Downarrow$

High temperature expansion

low temperature expansion:

$$Z = 2\, e^{K\cdot 2L^2} + 2\, e^{K(2L^2 - \text{Flip})} + \cdots$$

$$= 2 e^{K\cdot 2L^2} \sum_{\text{loop}} e^{-2K\cdot(\underline{\text{flipped bonds}})}$$



$$= e^{K\cdot 2L^2} \sum_{\substack{\text{closed}\\ \text{domain}}} e^{-2K\cdot \text{Length of domain}}.$$

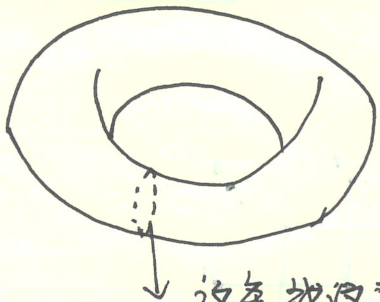
$$\cancel{= 2 e^{K\cdot 2L^2} \sum_{\text{closed}} \left(e^{-2K^2}\right)^{\text{Length}}}$$

$$Z = (2\operatorname{ch}K)^{2L^2} \sum_{\text{closed}} (\operatorname{th}K)^{\text{Length}}$$

$$= 2\cdot e^{K\cdot 2L^2} \sum_{\text{closed}} \left(e^{-2K^2}\right)^{\text{Length}}$$

if $\tanh K^* = e^{-2K^*}$

· 考虑 partition function，如果在 K 上是相变点，则在 K′ 也是相变点。



这条线没有将 torus 分成两个

封闭曲线不一定是 domain wall

· Ising model on torus:

$$Z(K) = \overset{*\,\leftarrow\,\text{Factor}}{\sum_{\text{closed}}} (\tanh K)^{\text{length}}$$

包含4项

- domain wall
- domain wall + loop 1
- domain wall + loop 2
- domain wall + loop 12

② 取对 anti-periodic condition 的 Ising model

$$Z_{PP}^{*K} = (*)\left(Z_{PP}^{K'} + Z_{PA}^{K'} + Z_{AP}^{K'} + Z_{AA}^{K'}\right)$$

周期边界条件 Ising model 做完 duality 是4种边界条件

for $\tanh K^* = e^{-2K^*}$

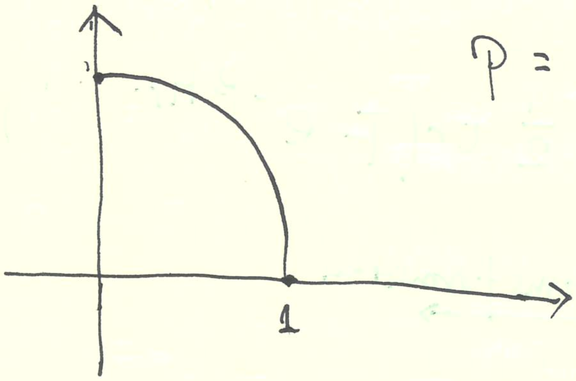
Duality in 1d Ising model:

$$H = -J\sum_{l}\sigma^z_l\sigma^z_{l+1} - h\sum_{j}\sigma^x_j$$

$$\tau^z_l = \sigma^z_l\sigma^z_{l+1}$$

$$\sigma^x_j = \tau^x_j\tau^x_{j-1}$$

$$= -J\sum\tau^z_l - h\sum_{l}\tau^x_{l-1}\tau^x_l$$



$P = \prod_{x}\sigma^x_i$. Z_2 symmetry

Hamiltonian have Z_2 symmetry, however the ground state have no Z_2 symmetry.

when: $J/h \to \infty$, $|\to,\to,\cdots\to\rangle$ structure stone Z_2 symmetry.

Jordan Wigner transformation

spin - Fermion : 边界条件改变

$$Z = \mathrm{Tr}(e^{-\beta H_p}) = \mathrm{Tr}\left(\frac{1+F}{2} e^{-\beta H_p}\right) + \mathrm{Tr}\left(\frac{1-F}{2} e^{-\beta H_p}\right)$$

$$\Big\downarrow \text{dual}$$

$$= \mathrm{Tr}\left(\frac{1+F}{2} e^{-\beta H_p(h,J)}\right) + \mathrm{Tr}\left(\frac{1+F}{2} e^{-\beta H_p(h,J)}\right)$$

$\Uparrow$ anti-symmetry :

$$= \frac{1}{2}\mathrm{Tr}\, e^{-\beta H_p(h,J)} + \frac{1}{2}\mathrm{Tr}\left(F e^{-\beta H_p(h,J)}\right)$$

$$+ \frac{1}{2}\mathrm{Tr}\left(e^{-\beta H_A(h,J)}\right) + \frac{1}{2}\mathrm{tr}\left(F e^{-\beta H_p \cdots}\right)$$

1+1d tranverse : $\xrightarrow{\text{Duality transformation}}$.

时间和空间 周期 反周期

Jordan Wigner transformation

Jiang sheng Han

- Find fixed point wavefunction $\longrightarrow$ TN

write down by symmetry

~~Sym~~

TN Eq $\longrightarrow$ order parameter

- 1D SPT phases:

simple cases: cluster phases:

$$H = -J \sum_j Z_{j-1} X_j Z_{j+1}$$

$$(Z_{j-1} X_j Z_{j+1}) |\Psi_j\rangle = (-1)^{\alpha_j} |\Psi_j\rangle$$

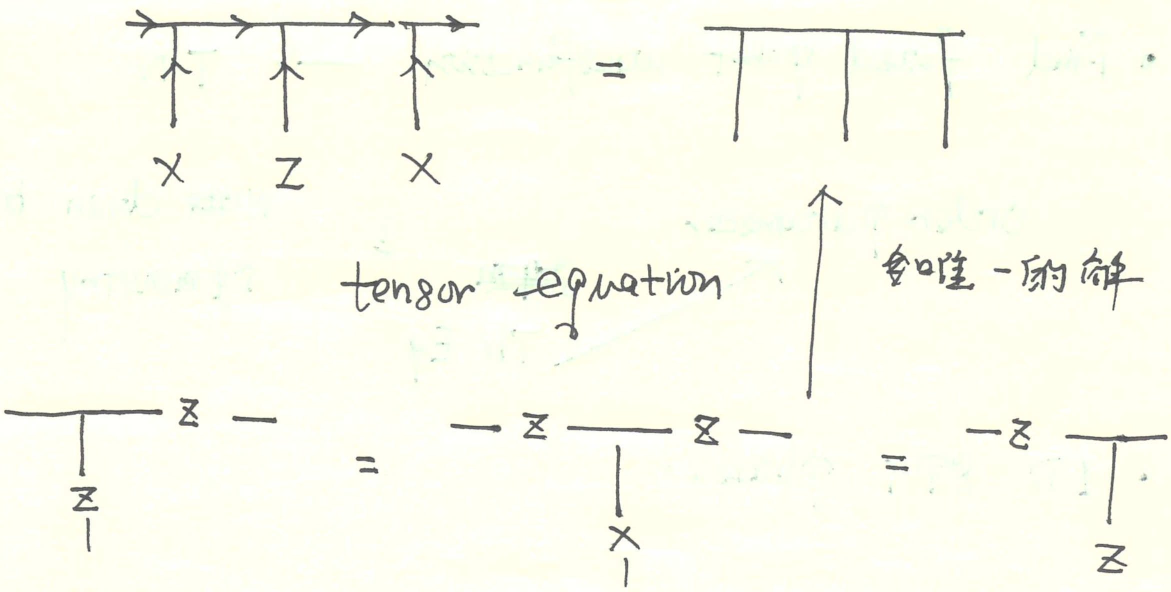
$$|\Psi\rangle = \bigotimes_{j=1}^{N} |\Psi_j\rangle$$

$Z_2^1 \otimes Z_2$ SPT

$\Downarrow$

$$\bigotimes_{j\in odd} X_j \equiv Z_2^1 \qquad Z_2^2 = \bigotimes_{j=even} X_j'$$

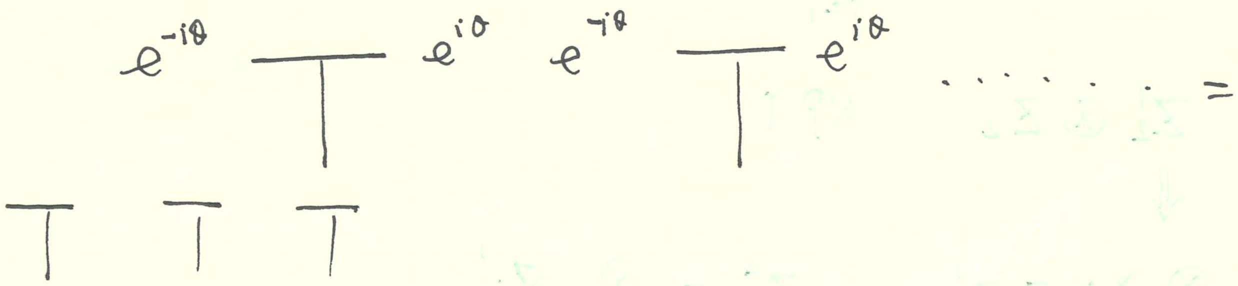
Fixed point wavefunctions:



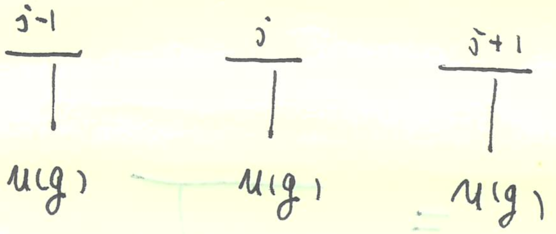
· string order

$$\lim_{l\to\infty}\langle\Psi|\prod_{j=1}^{l}u_j(g),\Psi\rangle$$

· Topological classification



Fixed point transformations:



$$(T_j)^a_{\alpha\beta}\ |a\rangle_j \otimes |\alpha\rangle_{j-1/2} \langle\beta|_{j+1/2}$$

$$\begin{cases} w(g)\,w(h)\,w(l) = \alpha(g,h)\,\alpha(gh,l)\,w(g,h,l) \\ w(g)\,w(h)\,w(l) = \alpha(l,h)\,\alpha(g,hl)\,w(g,h,l) \end{cases}$$

cocycle condition:

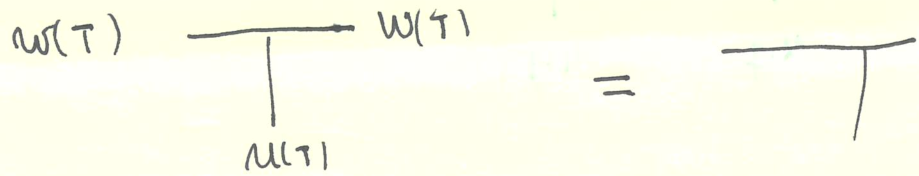
$$\alpha(g,h)\,\alpha(gh,l) = \alpha(l,h)\,\alpha(g,hl)$$

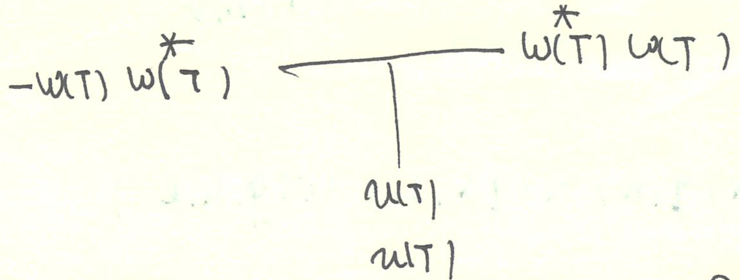
$\Downarrow$ 2-cocycle condition

coboundary: $\dfrac{\beta(g)\beta(h)}{\beta(gh)} \triangleq$ 2 coboundary

$B^2(G, U(1))$.

- Example: $Z_2 \otimes Z_2$





$\mathbb{Z}_2$ group 中不存在 SPT phase

$$\omega(T)\,\omega(\overset{*}{T}) = \omega(T,T)$$

$$\underbrace{\omega(T)\,\omega(\overset{*}{T})}\,\omega(T) = \omega(T,T)\,\omega(T)$$

$$\omega(T)\,\underbrace{\omega(\overset{*}{T})\,\omega(T)} = \omega(T)\,\omega(\overset{*}{T},T).$$

$$\Rightarrow \quad \omega(T,T) = \omega(\overset{*}{T},T)\,; \quad \Rightarrow \quad \omega(T,T) = \pm 1$$

Double degeneracy: